

Fuzzy

A Fuzzy set ~~A~~ $A \subseteq \mathbb{R}^n$ (A_α α -cut set)
is Convex if and only if:-

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_A(x_1), \mu_A(x_2)]$$

For all $x_1, x_2 \in \mathbb{R}^n$

الحل

Convex \iff الشرط يتحقق

\Rightarrow Let A_α is Convex for all.

$$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha \quad 0 \leq \lambda \leq 1$$

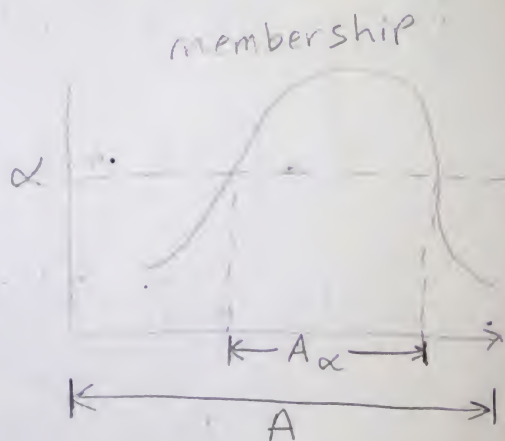
$$\Rightarrow \mu_A(x_1) \geq \alpha ; \mu_A(x_2) \geq \alpha$$

$$1(\lambda) + 2(1-\lambda)$$

$$\therefore \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha$$



$$0 \leq \lambda \leq 1$$



1 Lec 7

$$\Rightarrow \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha = \min(\mu_A(x_1), \mu_A(x_2))$$

$$\underline{\text{if}} \mu_A(x_1) \leq \mu_A(x_2)$$

إلا قِجَاه العكس

$$\underline{\text{Let}} \mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\mu_A(x_1), \mu_A(x_2)]$$

$$A_\alpha \neq \emptyset$$

$$\underline{\text{if}} \alpha = \mu_A(x_1) \leq \mu_A(x_2)$$

$$\Rightarrow \mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\alpha, \mu_A(x_2)]$$

$$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha$$

$\therefore A_\alpha$ is Convex

Magnitude of Fuzzy set

[1] Scalar Cardinality :-

← يمثل هذا المقياس مجموع درجات الإلتقاء لجميع عناصر الفترة.

$$|A| = \sum_{x \in A} \mu_A(x)$$

[2] Lec 7

2] Relative Cardinality:-

$$||A|| = \frac{|A|}{|X|} = \frac{\sum_{x \in A} \mu_A(x)}{\text{no. of element of } X}$$

← هذا المقياس يمثل مقدار التأكد من صدقية الـ (data) بالنسبة للعينة التي تجعل الفئة فئة فارغة.

Ex: Consider the fuzzy set: short, medium, tall

cm	short	middle	tall
14	1	0	0
15	1	0	0
16	0.9	0.1	0
17	0.7	1	0
18	0.3	0.8	0.3
19	0	0	1

1] Compare the support of each set

2] Compare the α -cut of each set at $\alpha=0.5$

3] |short| and ||short||

Solution

[1]

$$\text{supp}(A) = \{x : \mu(x) > 0\}$$

$$\text{supp}(\text{short}) = \{14, 15, 16, 17, 18\}$$

$$\text{supp}(\text{medium}) = \{16, 17, 18\}$$

$$\text{supp}(\text{tall}) = \{18, 19\}$$

[2] $A_\alpha = \{x : \mu(x) > \alpha\}$

$$(\text{short})_{0.5} = \{14, 15, 16, 17\}$$

$$(\text{medium})_{0.5} = \{17, 18\}$$

$$(\text{tall})_{0.5} = \{19\}$$

[3]

$$|\text{short}| = 1 + 1 + 0.9 + 0.7 + 0.3 = 3.9$$

$$\|\text{short}\| = \frac{3.9}{6}$$

[4] Lec 7

→ operation on Fuzzy set:-

① Complement:-

$$\mu_{A^c}(x) = 1 - \mu_A(x) \quad x \in X$$

② union:-

$$\mu_{A \cup B}(x) = \max \{ \mu_A(x), \mu_B(x) \}$$

③ intersection:-

$$\mu_{A \cap B}(x) = \min \{ \mu_A(x), \mu_B(x) \}$$

EX Let $\tilde{A} = \frac{0.3}{1} + \frac{0}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$

$\tilde{B} = \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$ Find

① \tilde{A}^c

② $\tilde{A} \cup \tilde{B}$

③ $\tilde{A} \cap \tilde{B}$

④ $\tilde{A} - \tilde{B}$

⑤ $\tilde{A} \Delta \tilde{B}$

Solution

$$\textcircled{1} \bar{A} = \frac{0.7}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.2}{4} + \frac{0}{5}$$

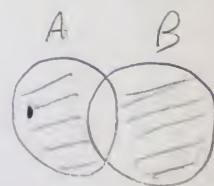
$$\textcircled{2} \tilde{A} \cup \tilde{B} = \frac{0.3}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$$

$$\textcircled{3} \tilde{A} \cap \tilde{B} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$$

$$\textcircled{4} \tilde{A} - \tilde{B} = \tilde{A} \cap \bar{\tilde{B}} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$$

$$\textcircled{5} \tilde{A} \Delta \tilde{B} = (\tilde{A} \cap \bar{\tilde{B}}) \cup (\bar{\tilde{A}} \cap \tilde{B}).$$

$$\tilde{A} \cap \bar{\tilde{B}} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$$



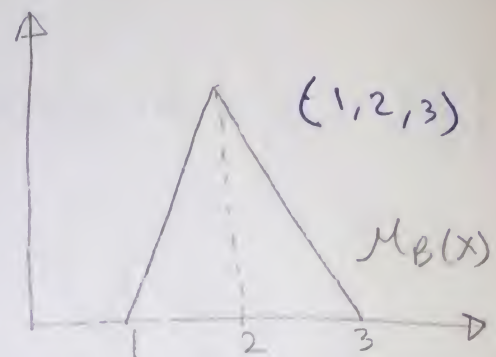
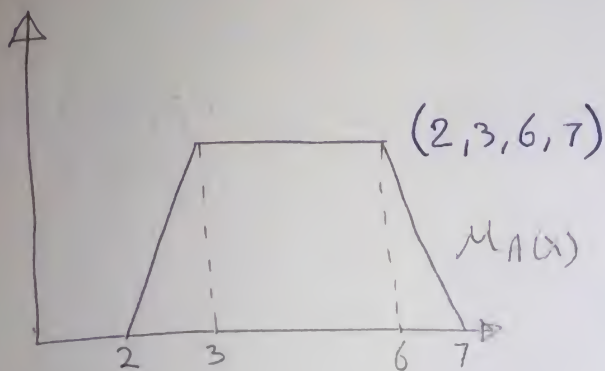
$$A \Delta B = (A - B) \cup (B - A)$$

$$\bar{\tilde{B}} \cap \tilde{A}$$

نفسه
↓
⋮
↓

← في الحالة المتصلة $\mu(x)$ متغير نقي
الفئة في الحالة (discrete)

[EX] Graphically represent the fuzzy set operation if the membership.



Find

1] $\mu_{\bar{A}}(x)$

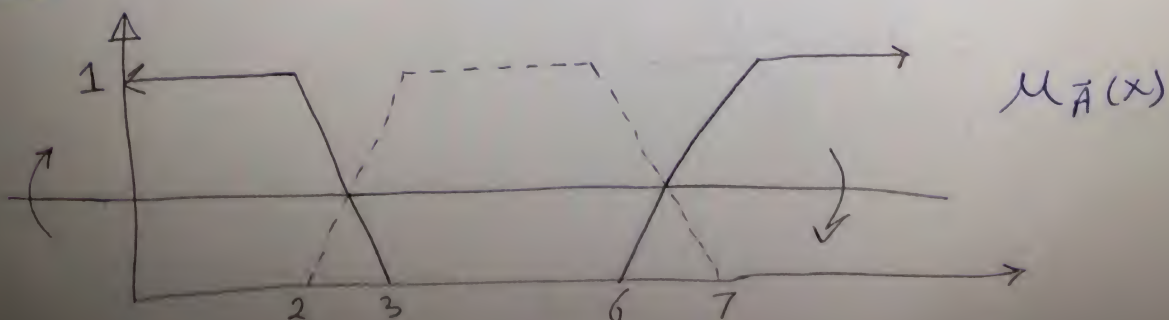
2] $\mu_{\bar{B}}(x)$

3] $\mu_{A \cup B}(x)$

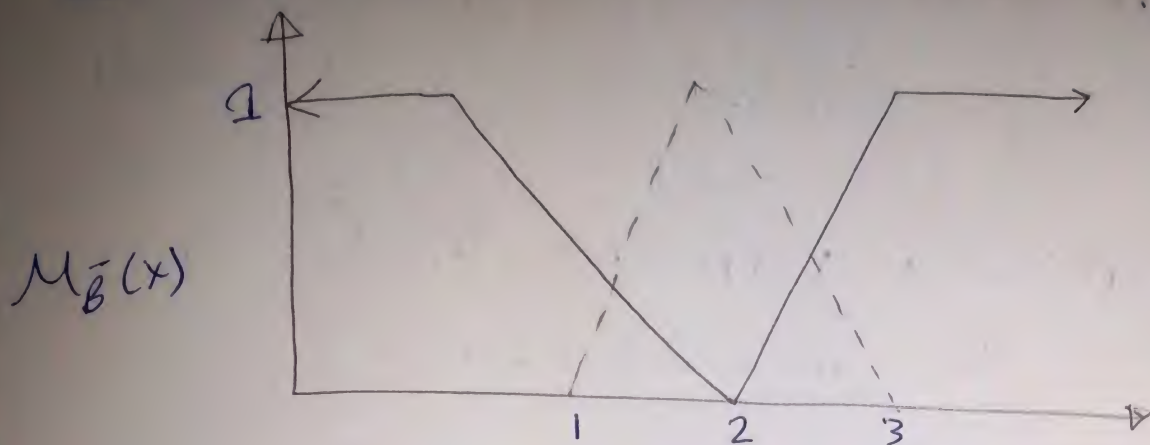
4] $\mu_{A \cap B}(x)$

Sol

1]

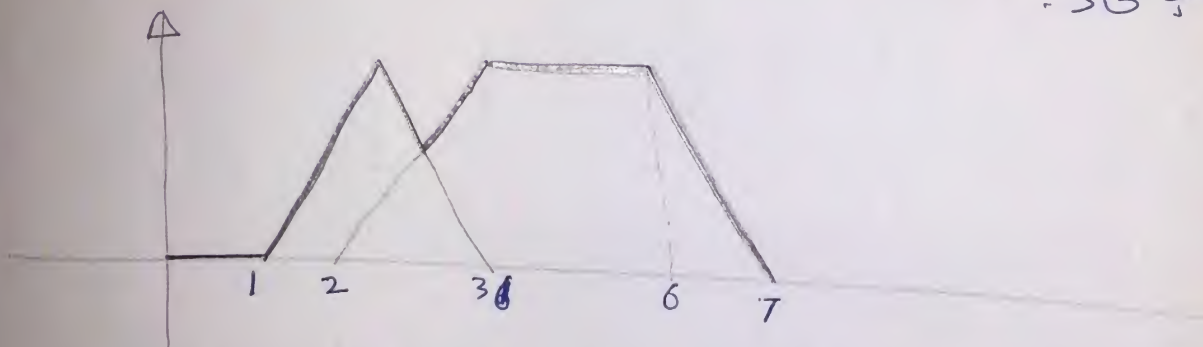


[2]



[3]

مع هزيم الرسمين خود بعض ونمش على الحرف الل فوق
في الاتحاد.



[4]

في التقاطع هزيم الرسمين على بعض ونمش على الحرف الل تحت.

